$pv = RT$

$R = 0.287$ kJ/kg K

$C_p = 1.005$ kJ/kg K

$C_v = 0.718$ kJ/kg K

Constant Volume Process

With no paddle work

$$Q = U_2 - U_1$$

when the fluid is a perfect gas this becomes

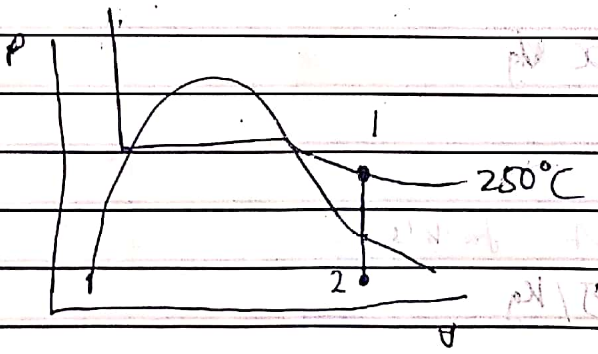
$$Q = C_v(T_2 - T_1)$$

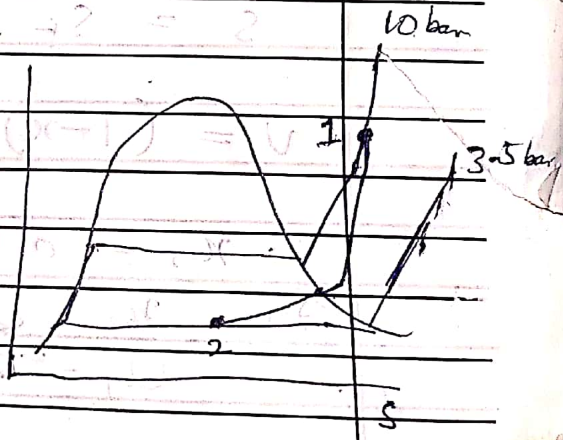
$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \text{ is}$$

$$S_2 - S_1 = \int_1^2 \left(\frac{dQ}{T}\right)_{rev} = \int_1^2 \frac{C_v}{T} dT = C_v \ln \frac{T_2}{T_1}$$

A fluid in a closed vessel of fixed volume $0.14 m^3$, exerts a pressure of 10 bar at 250 °C. If the vessel is cooled so that the pressure falls to 3.5 bar, determine the final temperature, heat transfer and change of entropy.

(a) Steam





Find the mass of the fluid in the system. Since T_1 is > T_s corresponding to P_1 the vapour is initially superheated. Using the superheat table we find

$$v_1 = 0.2328 m^3/kg$$

and since $V = 0.14 m^3$

$$m = \frac{V}{v_1} = \frac{0.14}{0.2328} = 0.6014 kg$$

$$m v_g = 0.6014 \times 0.5241 = 0.3152 m^3$$

which is > $0.14 m^3$

Therefore the vapour must be wet in its final state

The temperature must therefore be the saturation temperature corresponding to P_2 i.e

$t_2 = 138.9$ °C

the dryness fraction

$$x_2 = \frac{V}{m V_{g_2}} = \frac{0.14}{0.3152} = 0.444$$

$$u = (1-x)u_f + x u_g$$
$$h = (1-x)h_f + x h_g$$
$$s = (1-x)s_f + x s_g$$

$$\left.\begin{aligned} u &= u_f + x u_{fg} \\ h &= h_f + x h_{fg} \\ s &= s_f + x s_{fg} \end{aligned}\right\}$$ when the difference columns are given in the tables

$$v = (1-x)v_f + x v_g$$

$\therefore x_2 = 0.443$

From the super-heat table:

$U_1 = 2711$ kJ/kg

$$U_2 = U_{f_2} + x_2 U_{fg_2} = 584 + 0.443(2549 - 584)$$
$$= 1457 \text{ kJ/kg}$$

$$\therefore Q = m(U_2 - U_1) = 0.6014(1457 - 2711)$$
$$= -754 \text{ kJ}$$

From the superheat tables $S_1 = 6.926$ kJ/kgK

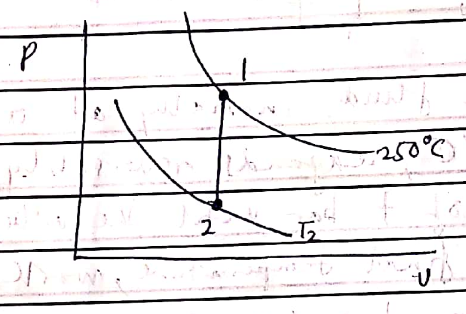
and from the saturation table:

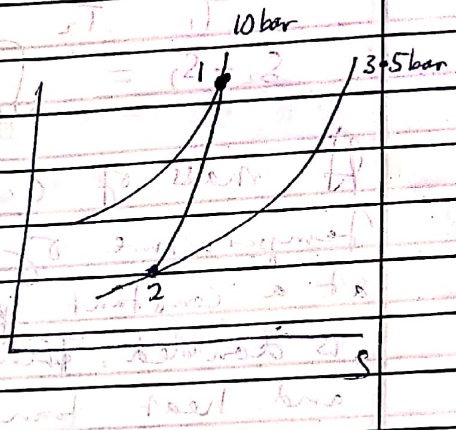
$$S_2 = S_{f_2} + x_2 S_{fg_2}$$
$$= 1.727 + (0.443 \times 5.214) = 4.042 \text{ kJ/kgK}$$

Change of entropy:

$$S_2 - S_1 = m(s_2 - s_1) = 0.6014(4.042 - 6.926)$$
$$= -1.734 \text{ kJ/K}$$

(b) Air





$$m = \frac{pV}{RT_1} = \frac{100 \times 10 \times 0.14}{0.287(250+273)} = 0.933 \text{ kg}$$

Final temperature:

$$T_2 = \frac{p_2}{p_1} T_1 = \frac{3.5}{10} \times 523 = 183 \text{K}$$

Heat Transfer

$$Q = mC_v(T_2 - T_1) = 0.933 \times 0.718(183 - 523)$$
$$= -228 \text{ kJ}$$

Change of entropy:

$$S_2 - S_1 = mC_v \ln\frac{T_2}{T_1} = 0.933 \times 0.718 \ln\frac{183}{523}$$
$$= -0.703 \text{ kJ/K}$$

Constant Pressure Process

For a reversible pressure process undergone by a unit mass of any fluid

$$W = -p\int_1^2 dv = -p(v_2 - v_1)$$
$$Q = (u_2 - u_1) - W = (u_2 - u_1) + p(v_2 - v_1) = h_2 - h_1$$

When the fluid is a perfect gas

$$W = -R(T_2 - T_1)$$
$$Q = c_p(T_2 - T_1)$$

For a perfect gas we also have the following relations between properties which are valid for any two states of the same pressur

$$\frac{v_1}{T_1} = \frac{v_2}{T_2}$$

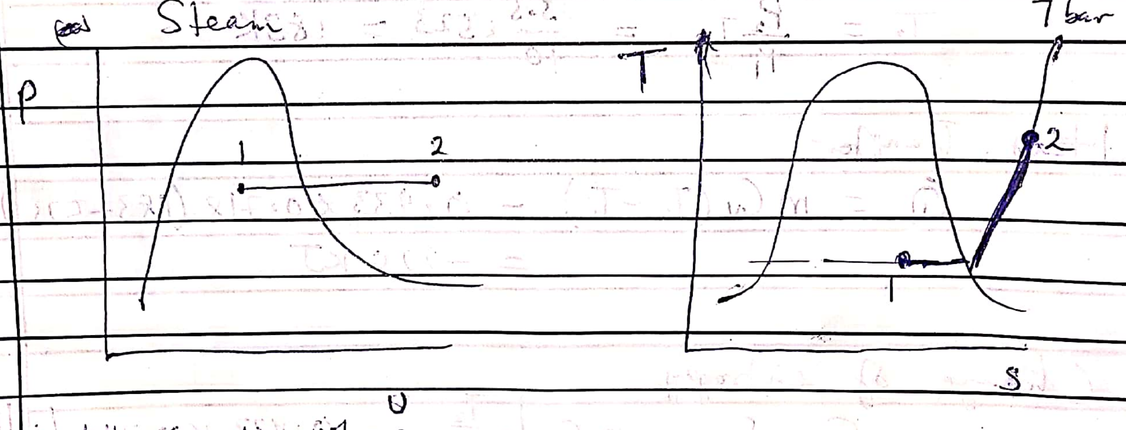
$$S_2 - S_1 = \int_1^2 \left(\frac{dQ}{T}\right)_{rev} = \int_1^2 \frac{C_p}{T}\,dT = C_p \ln \frac{T_2}{T_1}$$

A mass of 0.2kg of fluid, initially at a temperature of 165 °C, expands reversibly at a constant pressure of 7 bar until the volume is doubled. Find the final temperature, work and heat transfers in two cases:

(a) when the fluid is steam with an initial dryness fraction of 0.7,

(b) when the fluid is air

(a) Steam



initial specific volume

$$v_1 = x_1 v_{g_1} = 0.7 \times 0.2728 = 0.191\ m^3/kg$$

final specific volume

$$v_2 = 2v_1 = 0.382\ m^3/kg$$

Since $v_2 > v_{g2}$, the vapour must be superheated in its final state.

From the superheat table by interpolation

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ither
rgin

T	T₂	300	T_2	350
V_2		0.3714	0.382	0.4058

Question...
Write on both sides of the paper

$$\frac{\cancel{350}-T_2}{\cancel{350-300}} = \frac{\cancel{0.4058}}{\cancel{0.4058-0.3714}}$$

$$\frac{T_2-300}{350-300} = \frac{0.382-0.3714}{0.4058-0.3714}$$

~~0.382~~

Do not write in either margin

The final temperature is

$$T_2 = 300 + \left(\frac{0.382 - 0.3714}{0.4058 - 0.3714}\right)50 \qquad T_2$$

$$= 315\,°C$$

Work Transfer:

$$_1W_1 = -mp(v_2 - v_1) = -0.2 \times 100 \times 7\,(0.382 - 0.191)$$

$$= -26.7\ KJ$$

From the saturation table:

$$h_1 = h_{f_1} + x_1 h_{fg_1} = 697 + (0.7 \times 2067) = 2144\ KJ/kg$$

From the superheat table by interpolation

$$h_2 = 3060 + \frac{15}{50}(3164 - 3060)$$

300	315	350
3060	h_2	3164

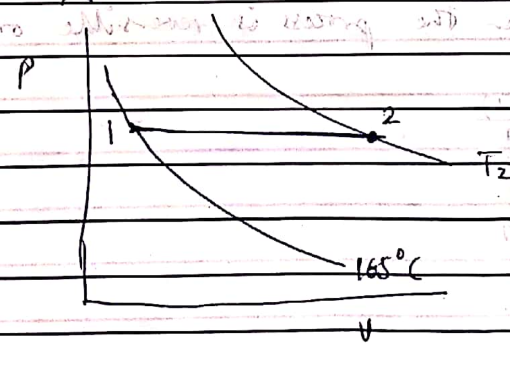
$$= 3091\ KJ/kg$$

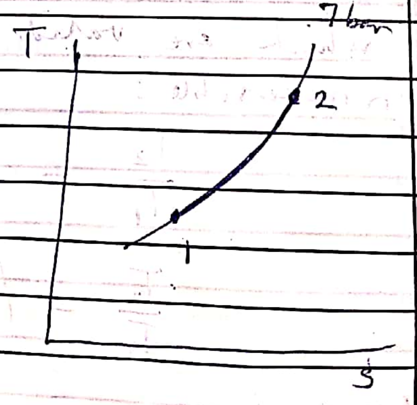
Heat transfer:

$$\cancel{Q = mC_p(T_2 - T_1) = 0}$$

$$Q = m(h_2 - h_1) = 0.2(3091 - 2144) = 189\ KJ$$

Air





Initial volume: $V_1 = \frac{mRT_1}{P_1} = \frac{0.2 \times 0.287 \times 438}{100 \times 7}$

$$= 0.03592\ m^3$$

Final volume:

$$V_2 = 2V_1 = 0.07184\ m^3$$

Do not write in either margin

$1\ \text{bar} = 10^5\ N/m^2$ ~~100~~

$1\ kN/m^2 = 10^2\ [\text{bar}]$

$1\ N/m^2 = \frac{1}{10^5}\ \text{bar}$

Question......................................
Write on both sides of the paper

Do not write in either margin

Final temperature : $T_2 = \frac{V_2}{V_1} T_1 = 876\ K$

Work transfer :

$$W = -p(V_2 - V_1) = -100 \times 7 \times 0.03592 = -25.1\ kJ$$

Heat transfer :

$$Q = mC_p(T_2 - T_1) = 0.2 \times \cancel{1000}\, 1.005 \times 438 = 88.0\ kJ$$

Polytropic Process

When a polytropic process is reversible the work transfer is

$$W = \frac{(P_2V_2 - P_1V_1)}{n-1}$$

Therefore, the from the energy equation,

$$Q = (U_2 - U_1) - \frac{(P_2V_2 - P_1V_1)}{n-1}$$

When the fluid is a perfect gas, these expressions can be written as

$$W = \frac{R}{n-1}(T_2 - T_1)$$

$$Q = C_v(T_2 - T_1) - \frac{R}{n-1}(T_2 - T_1) = \left(C_v - \frac{R}{n-1}\right)(T_2 - T_1)$$

since $p_1v_1^n = P_2v_2^n$ and $pv = RT$, we also have the following relations between properties which are valid whether the process is reversible or irreversible :

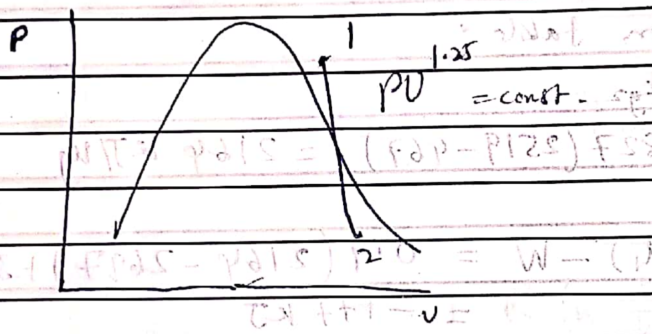
$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(n-1)}{n}}$$

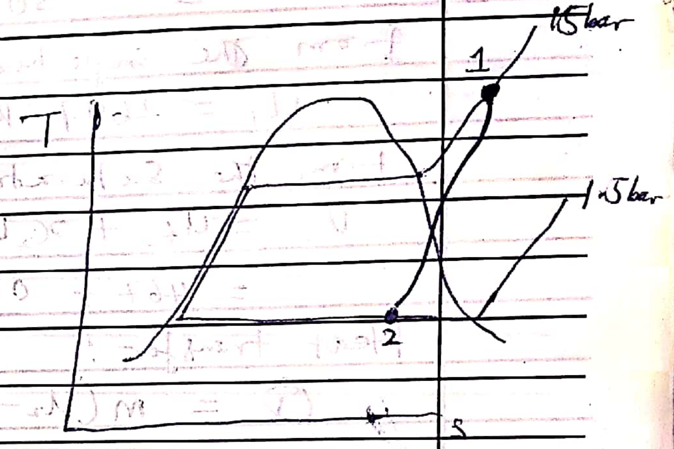
$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1-n}$$

(The change of entropy is best found from first principles as will be shown in the following example)

A mass of 0.9 kg of fluid, initially at a pressure of 15 bar and a temperature of 250 °C expands reversibly and polytropically to 1.5 bar. Find the final temperature, work and heat transfers and change of entropy; if the index of expansion is 1.25

Steam





From the superheat table:

$v_1 = 0.152\ m^3/kg$ ✓

Final specific volume:

$$v_2 = v_1 \left(\frac{P_2}{P_2}\right)^{\frac{1}{n}} = 0.1520 \times 10^{1/1.25} = 0.959\ m^3/kg$$

V_{g_2} is 1.159 m^3/kg so that the vapour is wet in the final state, and the temperature is the saturation value corresponding to 1.5 bar, i.e. ~~T_2 = 111.4~~

$T_2 = 111.4$ °C

P_2

$$v_2 = (1-x)\,V_{f_2} + x\,V_{g_2}$$

$$0.959 = (1-x)$$

V_g liquid

$x = 1$

Dryness fraction

$$x_2 = \frac{v_2}{V_{g_2}} = \frac{0.959}{1.159} = 0.827$$

T.	T_2	300	T_2	350
V_2		0.3714	0.382	0.4058

$\frac{T_2 - 300}{350 - 300} = \frac{0.382 - 0.3714}{~~0.382~~\ 0.4058 - 0.3714}$

~~$\frac{300 - T_2}{350 - 300} = \frac{0.4058}{0.4058 - 0.3714}$~~

The final temperature is

$$T_2 = 300 + \left(\frac{0.382 - 0.3714}{0.4058 - 0.3714}\right)50 \qquad T_2$$

$= 315°C$

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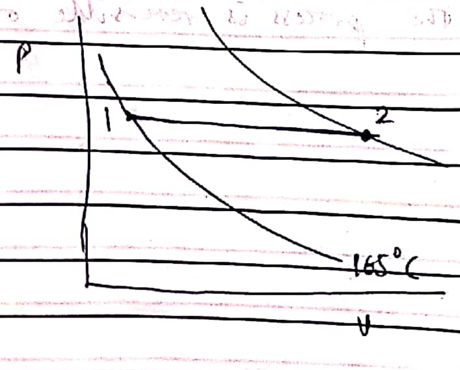
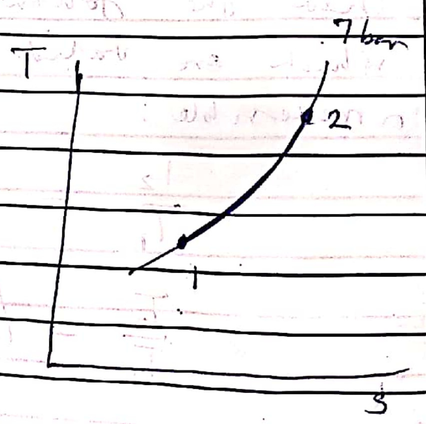
Air





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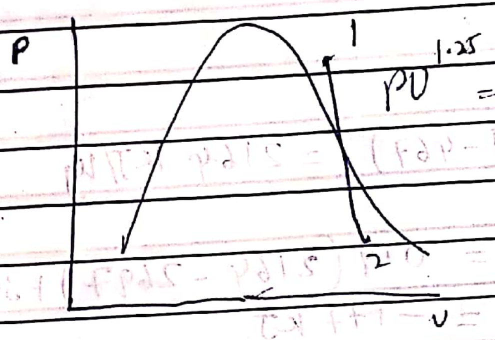
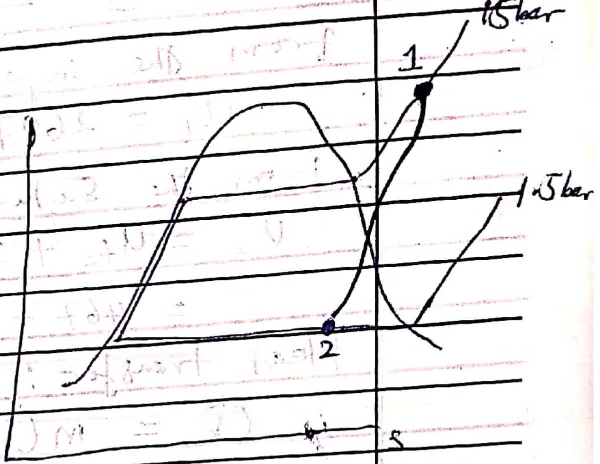
Steam





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v_{g_2} is 1.159 m^3/kg so that the vapour is wet in the final state, and the temperature is the saturation value corresponding to 1.5 bar, i.e. ~~T_2 = 111.4~~

$T_2 = 111.4$ °C

P_1

$$\boxed{\begin{aligned} v_2 &= (1-x)v_{f_2} + x v_{g_2} \\ 0.959 &= (1-x) \end{aligned}}$$

v_g Liquid
$x = 1$

Dryness fraction

$$x_2 = \frac{v_2}{v_{g_2}} = \frac{0.959}{1.159} = 0.827$$

Work transfer:

$$W = \frac{m}{n-1}\left(P_2V_2 - P_1V_1\right)$$

$$= \frac{100 \times 0.9}{0.25}\left\{(1.5\times 0.959)-(15\times 0.152)\right\}$$

$$= -303 \text{ KJ}$$

From the superheat table:

$$u_1 = 2697 \text{ KJ/Kg}$$

From the saturation table:

$$u_2 = u_{f2} + x_2 u_{fg2}$$

$$= 467 + 0.827(2519-467) = 2164 \text{ KJ/Kg}$$

Heat transfer:

$$Q = m(u_2-u_1) - W = 0.9(2164-2697)+303$$

$$= -177 \text{ KJ}$$

$$S_1 = 6.711 \text{ KJ/KgK}$$

$$S_2 = S_{f2} + x_2 S_{fg2} = 1.434 + (0.827 \times 5.789)$$

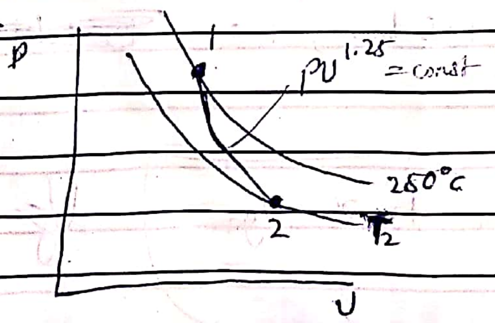
$$= 6.222 \text{ KJ/KgK}$$

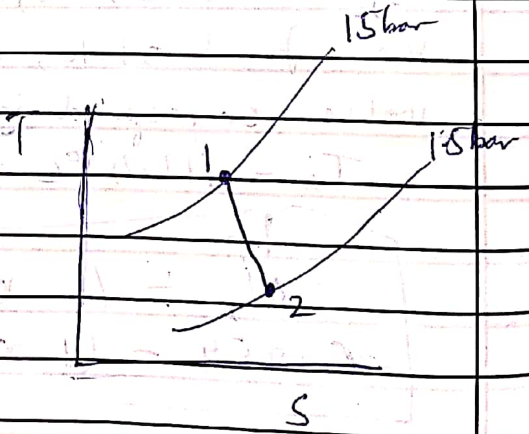
Change of entropy:

$$S_2 - S_1 = m(S_2 - S_1) = 0.9(6.222 - 6.711)$$

$$= -0.440 \text{ KJ/K}$$

Air





Final temperature:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{(n-1)/n} = 523\left(\frac{1}{10}\right)^{0.25/1.25}$$

$$= 330 \text{ K}$$

Work Transfer :

$$W = \frac{mR}{n-1}(T_2 - T_1) = \frac{0.9 \times 0.287}{0.25}(330 - 523)$$

$$= -199 \text{ kJ}$$

Heat transfer :

$$Q = mC_v(T_2 - T_1) - W$$

$$= \{0.9 \times 0.718(330 - 523)\} + 199 \quad = 74.3 \text{kJ}$$

Change of entropy, from first principles :

$$dS = \left(\frac{dQ}{T}\right)_{rev} = \frac{1}{T}du + \frac{P}{T}dV = \frac{C_v}{T}dT + \frac{R}{V}dV$$

(for perfect gas)

$$S_2 - S_1 = C_v \ln\frac{T_2}{T_1} + R\ln\frac{V_2}{V_1}$$

$$= C_v \ln\frac{T_2}{T_1} - \frac{R}{n-1}\ln\frac{T_2}{T_1} \quad \left[\text{using } \frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{1-n}\right]$$

$$= \left(C_v - \frac{R}{n-1}\right)\ln\frac{T_2}{T_1} \quad \left[\frac{V_2}{V_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{1-n}}\right]$$

$$C_v \ln\frac{T_2}{T} + R\ln\left(\frac{T_2}{T_1}\right)^{\frac{1}{1-n}}$$

$$C_v \ln\frac{T_2}{T_1} + \frac{R}{1-n}\ln\frac{T_2}{T_1}$$

$$C_v \ln\frac{T_2}{T_1} - \frac{R}{n-1}\ln\frac{T_2}{T_1}$$

$$\left(C_v - \frac{R}{n-1}\right)\ln\frac{T_2}{T_1}$$

$$S_2 - S_1 = 0.9\left(0.718 - \frac{0.287}{0.75}\right)\ln\frac{330}{523} = 0.178\,kJ/K$$

Adiabatic process

For a fluid undergoing an adiabatic process the energy equation per unit mass reduces to

$$W = (u_2 - u_1)$$

when the fluid is a perfect gas,

$$W = C_v(T_2 - T_1)$$

For a perfect gas undergoing a reversible adiabatic (i.e. isentropic) process, we can deduce the following relations between properties;

$dQ = 0$

$$0 = C_v dT + p dv \quad \text{(First Law)}$$

$$R dT = p dv + v dp \quad \text{(equation of state)}$$

Eliminating dT from these two equations, we obtain

$$0 = \left(1 + \frac{C_v}{R}\right) p dv + \frac{C_v}{R} v dp$$

$$= C_p p dv + C_v v dp \quad (\text{since } R = C_p - C_v)$$

Writing $\frac{C_p}{C_v} = \gamma$, this reduces to

$$\gamma \frac{dv}{v} + \frac{dp}{p} = 0$$

and on integration,

$$\gamma \ln v + \ln p = \text{constant}$$

$$pv^{\gamma} = \text{constant}$$

Since we also have $\frac{pv}{T}$ = constant, we can eliminate v and p in turn to give

$$Tp^{(1-\gamma)\gamma} = \text{constant}$$

or $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

φ

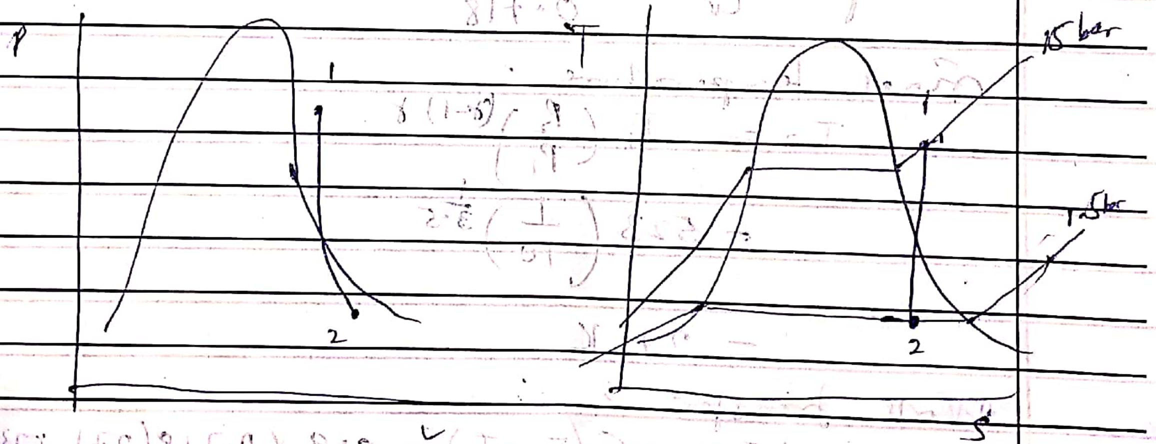
$T v^{\gamma-1}$ = constant

or $\dfrac{T_2}{T_1} = \left(\dfrac{v_2}{v_1}\right)^{1-\gamma}$

It is ~~cespe~~ evident from pv^γ = constant that, For a perfect gas, the reversible adiabatic process is a special case of the reversible polytropic process γ is called the index of isentropic expansion or compression

re-state the last example putting adiabatically in place of polytropically.
Find the final temperature and the work done

Steam



To find the final state we make use of the fact that $s_2 = s_1$ and from the superheat table, $s_1 = 6.711$ kJ/kg K.

Since $s_1 = s_2 < s_{g_2}$ the vapour must be wet in the final state

$\therefore T_2 = 111.4°C$ (i.e. T_s at P_2)

from
$S_1 = S_2 = S_{f2} + x_2 S_{fg2}$ we have

$$x_2 = \frac{S_2 - S_{f2}}{S_{fg2}} = \frac{6.711 - 1.434}{5.789} = 0.912$$

From the superheat table:

$U_1 = 2697$ KJ/kg

From the saturation table:

$U_2 = U_{f2} + x_2 U_{fg2}$
$= 467 + (0.912 \times 2052) = 2338$ KJ/kg

Work transfer:

$W = m(U_2 - U_1) = 0.9(2338 - 2697)$
$= -323$ KJ

Air

$$\gamma = \frac{C_p}{C_v} = \frac{1.005}{0.718} = 1.40$$

final temperature:

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(\gamma-1)\gamma}$$

$$= 523 \left(\frac{1}{10}\right)^{\frac{1}{3.5}}$$

$= 271$ K

Work transfer:

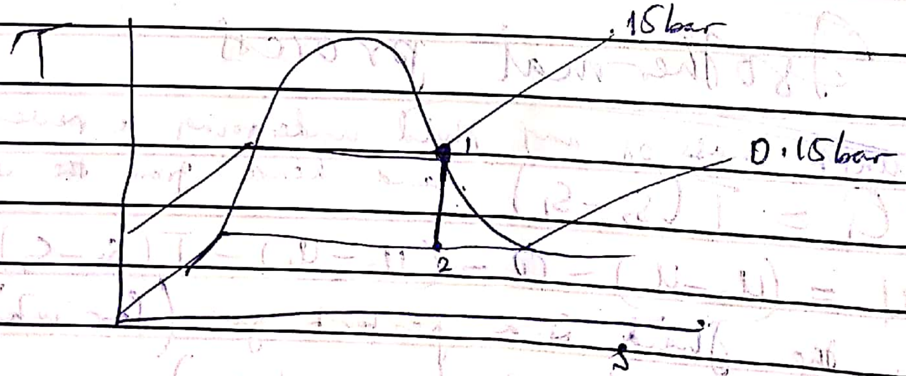
$W = mC_v(T_2 - T_1) = 0.9 \times 0.718(271 - 523)$
$= -163$ KJ

For a vapour, there is no simple and unique relation between p and v which applies to every isentropic expansion and compression. Nevertheless it is sometimes convenient to assume an approximate relation of the polytropic form pv^n = constant; n then becomes the index of isentropic expansion or compression, although it is not necessarily equal to the ratio c_p/c_v. Indeed, if part of the process occurs in the wet region, where $c_p = \infty$ this ratio has no meaning. The value of n can be found, ~~as in the following example~~ by substituting the values of p and v at the end states in the expression

$$p_1 v_1^n = p_2 v_2^n$$

6)

Steam, initially dry saturated, expands isentropically from a pressure of 15 bar to 0.15 bar. Find the index of isentropic expansion.



$$s_2 = s_1 = s_{g_1} = 6.445 \text{ KJ/kgK}$$

$$x_2 = \frac{s_2 - s_{f2}}{s_{fg2}} = \frac{6.445 - 0.755}{7.254} = 0.784$$

$$v_1 = v_{g_1} = 0.1317 \text{ m}^3/\text{kg}$$

$v_2 = x_2 v_{g2} = 0.784 \times 10.06 = 7.887\ m^3/kg$

If we can write $p_1 v_1^n = p_2 v_2^n$, then

$15 \times 0.1317^n = 0.15 \times 7.887^n$

from which $n = 1.125$

$\log 15 + n \log 0.1317 = \log 0.15 + n \log 7.88$

$1.1761 + n(-0.880) = -0.8239 + n(0.8965)$

$1.1761 + 0.8239 = 0.8965n + 0.880n$

$2 = 1.7765n$

$n = \frac{2}{1.7765} = 1.12587$

This is the approximate method of treating the isentropic expansion or compression of a vapour as a polytropic process.

Isothermal process

For unit mass of any fluid undergoing a reversible isothermal process, $Q = T(s_2 - s_1)$ and hence from the energy equation

$W = (u_2 - u_1) - Q = (u_2 - u_1) - T(s_2 - s_1)$

When the fluid is a perfect gas (for which u is a function ~~of~~ only of temperature), $u_2 = u_1$ and it follows from the energy equation that $Q = -W$

making use of the equation of state we can find W from $\int p\,dv$ if the process is reversible. Thus for a reversible isothermal process undergone by a perfect gas,

$$Q = -W = \int_1^2 p\,dv = RT\int_1^2 \frac{dv}{v} = RT\ln\frac{v_2}{v_1}$$

For a perfect gas we also have the following relations between properties which are valid for any two states of the same temperature;

$$p_1v_1 = p_2v_2$$

$$s_2 - s_1 = \frac{1}{T}\int_1^2 (dQ)_{rev} = R\ln\frac{v_2}{v_1}$$

Since $p_1v_1 = p_2v_2$ we can write $Q = -W = RT\ln\left(\frac{v_2}{v_1}\right)$

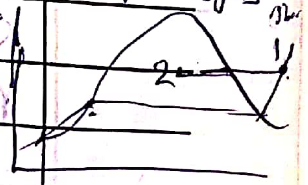
alternatively as: $Q = -W = -RT\ln\left(\frac{p_2}{p_1}\right)$

and hence also

$$s_2 - s_1 = -R\ln\frac{p_2}{p_1}$$

Evidently, ~~for~~ a reversible isothermal process is a special case of a reversible polytropic process, i.e. one for which $n = 1$.

A fluid, initially at 155.5 °C and 1 bar, is compressed reversibly and isothermally to a state where ~~a~~ the specific volume is 0.28 m³/kg. Find the change ~~in~~ of internal energy, change of entropy, and heat and work transfer per kg of fluid;



Steam

From the superheat table, by interpolation:

$$u_1 = 2583 + \frac{5.5}{50}76 = 2591\ \text{kJ/kg}$$

Since $v_2 = 0.28$ m³/kg is less than the specific volume of saturated vapour at 155.5 °C the vapour must be wet in its final state.

Hence $x_2 = \frac{v_2}{v_{g_2}} = \frac{0.28}{0.3427} = 0.817$

$u_2 = u_{f_2} + x_2 u_{fg_2} = 655 + (0.817 \times 1910) = 2215$ kJ/kg

Change of internal energy:

$u_2 - u_1 = 2215 - 2591 = -376$ kJ/kg

from the superheat table, by interpolation:

$s_1 = 7.614 + \frac{5.5}{50}\,0.220 = 7.638$ kJ/kg K

also:

$s_2 = s_{f_2} + x_2 s_{fg_2} = 1.897 + (0.817 \times 4.893)$
$= 5.895$ kJ/kg

Change of entropy:

$s_2 - s_1 = 5.895 - 7.638 = -1.743$ kJ/kg K

Heat transfer:

~~$W = (u_2 - u_1) - Q$~~

$Q = T(s_2 - s_1) = 428.5\,(-1.743)$
$= -747$ kJ/kg

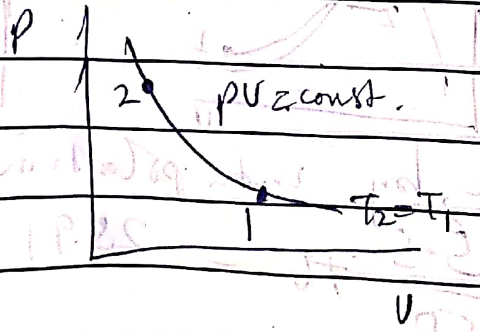
Work transfer:

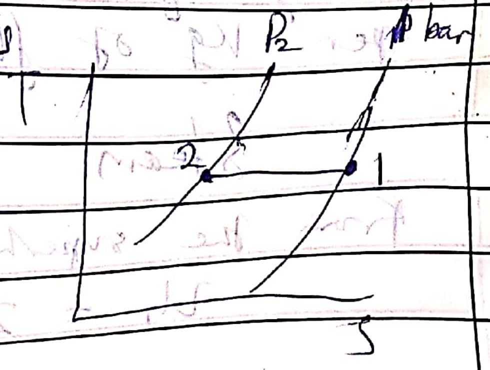
$W = (u_2 - u_1) - Q = -376 + 747$
$= 371$ kJ/kg

Ans

Change of internal energy is zero

Initial specific volume v





$v_1 = \frac{RT}{p_1} = \frac{0.287 \times 428.5}{100 \times 1.0} = 1.230\ \text{m}^3/\text{kg}$